# RESONANT SCATTERING OF AN ELASTIC WAVE BY A LAYER CONTAINING A RANDOM OR PERIODIC DISTRIBUTION OF INCLUSIONS

V. K. Kinra, K. Maslov, B. K. Henderson, N. Day, and G. Diderich
Department of Aerospace Engineering
Texas A & M University, College Station, Texas 77843-3141

## INTRODUCTION

Wave propagation through particulate composites has received considerable attention in recent years. The dispersive wave propagation through particulate composites with both random and periodic distributions has been studied theoretically [1] and experimentally [2-7]. The response of a layered composite with a finite number of layers can be predicted using the acoustic complex-valued transfer functions for a single layer [8]. The effect of the in-plane structure of an inclusion layer and resonance of individual particles on the wave propagation phenomena have been studied [9]. For a single layer of inclusions, it was shown that the arrangement of the inclusions has a significant effect on a wave propagating normal to the layer. The objective of this work is to study further the effect of the in-plane structure of an inclusion layer and acoustical properties of individual particles on the wave propagation phenomena.

## EXPERIMENTAL PROCEDURES

### Apparatus

A water-immersion apparatus which was used to conduct the experiments was described in [9]. The transmitters and receivers are matched pairs of Panametrics broadband, piezoelectric transducers with center frequencies ranging from 0.5 MHz to 2.25 MHz and a crystal diameter of 25.4 mm. A short-duration pulse is applied to the transmitter by a Panametrics Pulser/Receiver (Model 5052UA). The received signal is preamplified by the pulser/receiver and then digitized by a Tektronix DSA 601 at a sampling interval of 2 ns and a record length of 16,384 points.

To determine the transmission transfer function two measurements were taken for each test. The first measurement was taken through a neat polyester specimen and the second was through a specimen containing a layer of inclusions normal to the direction of wave propagation. The transfer function of the layer of spheres immersed in polyester, $H^*(\omega)$, is found by dividing the FFT of the transmitted or reflected waves, by the FFT of that for the neat sample.

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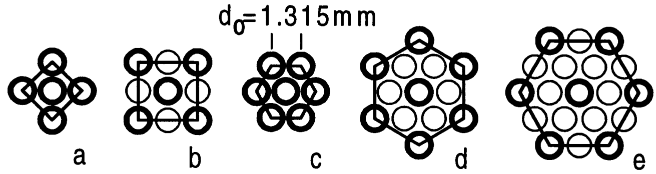


Figure 1. Particle arrangement within the layer.

## Specimens Preparation

Procedure of the specimen preparation is described in detail in [9,10]. A polyester casting resin hardened by a curing agent was chosen as the matrix material for this study. Specimens were manufactured using either lead (1.2 mm diameter, 10.1 mg mass), steel (1.17 mm, 6.6 mg), or glass (1.2 mm, 1.85 mg) spheres using steel molds 50.8 mm in diameter with 0.5 mm diameter holes drilled in square and hexagonal arrays with a distance $d_0$=1.315 mm between holes. Fully-packed specimens were made with every lattice site in the grid occupied as shown in Fig. 1a and Fig. 1c. The area fraction of a layer is defined as the projected area of the particles normalized by the total cross-section area of the specimen cylinder. A fully-packed square specimen has an area fraction of 0.64, while a fully-packed hexagonal square specimen has an area fraction of 0.72. In addition to the fully-packed specimens, square array specimens were made with a particles spaced by $d = \sqrt{2} \cdot (1.315)$ mm as shown in Fig. 1c and hexagonal array specimens were made with a particles spaced by $d = \sqrt{3} \cdot (1.315)$ mm and $d = 2 \cdot (1.315)$ mm as shown in Fig. 1d and Fig.1e, respectively. In order to isolate the effects that were due to the periodic arrangement of the particles, a number of single layer specimens were manufactured with a random arrangement. One random counterpart was made for each periodic specimen. The correct number of spheres were weighed so that the projected area fraction was equal to that of its counterpart. All specimens were cut to 7 cm length perpendicular to the axis of the cylinder and polished.

## Acoustical Properties of the Constituents

To determine the properties of the neat polyester, reference specimens without inclusions were tested using the technique detailed in [9,12]. The phase velocity and density were found to differ by less than 1% between samples. The attenuation was found to vary by as much as 20% between samples. The measured attenuation increase linearly with the frequency in the frequency range 0.2 MHz - 4 MHz. The geometry and the size of the spheres made it difficult to measure the acoustic properties of their material. Therefore, the acoustic properties of spheres material are taken from [4,13]. The acoustic properties of the constituents are presented in Table I.

# RESULTS

## Random Distribution of Inclusions

Figures 2, 3, and 4 show respectively, the magnitude of the transfer function for 0.24 area fraction layer of a randomly distributed lead, stainless steel, and glass particles. Variation of the experimentally measured transfer function with frequency in case of

Table 1. Acoustic properties of the constituents.

| Material | $C_L$ (mm/μs) | $C_S$ (mm/μs) | ρ (g/cm$^3$) | α (nepers/mm) |
|---|---|---|---|---|
| Polyester | 2.49 | 1.18 | 1.22 | 0.017 @ 1 MHz |
| Lead | 2.21 | 0.86 | 11.3 | 0.026 @ 2 MHz |
| Steel | 5.94 | 3.2 | 7.8 | negligible |
| Glass | 5.66 | 3.3 | 2.49 | negligible |

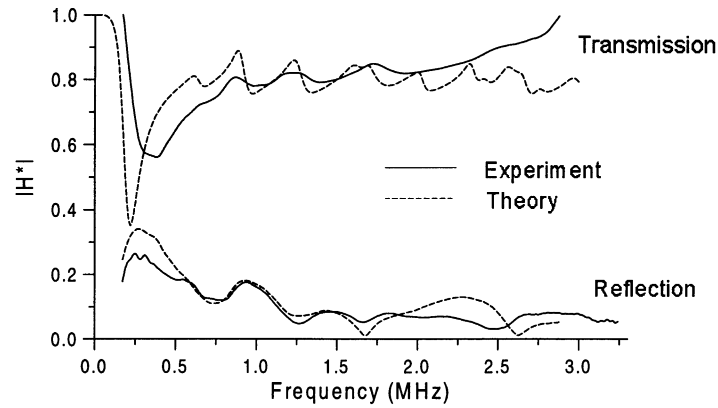


Figure 2. Transfer function for the random single layer lead specimen.

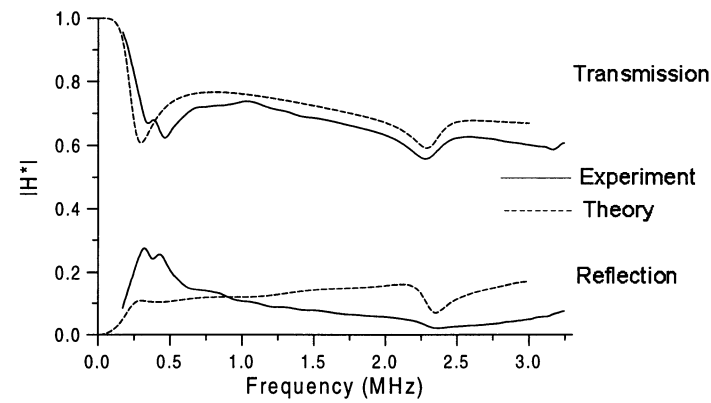


Figure 3. Transfer function for the random single layer steel specimen.

reflection can be partially explained by the variation of the back-scattering from a single sphere. Ying and Truell [14] solution of the back-scattering amplitude is plotted in Figures 2-4. Theoretical and experimental curves qualitatively coincide. In case of through-transmission the variation of transfer function magnitude is relatively small. It can be attributed to the variation of total scattering cross-section for a plane longitudinal wave transmitted through the layer of inclusions.

Samples with glass particles and with steel particles display resonance at 2.6 MHz and 2.3 MHz, respectively. Samples with lead particles and with steel particles show rigid

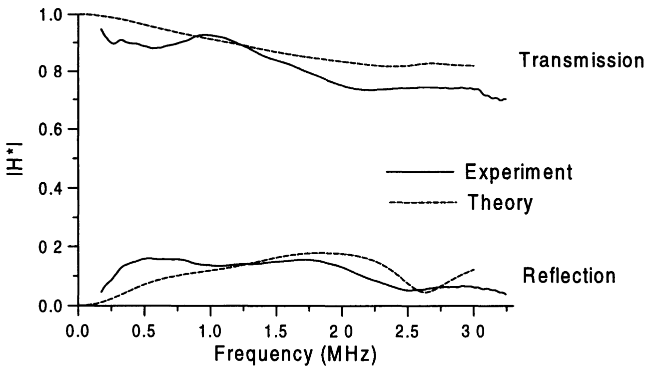


Figure 4. Transfer function for the random single layer glass specimen.

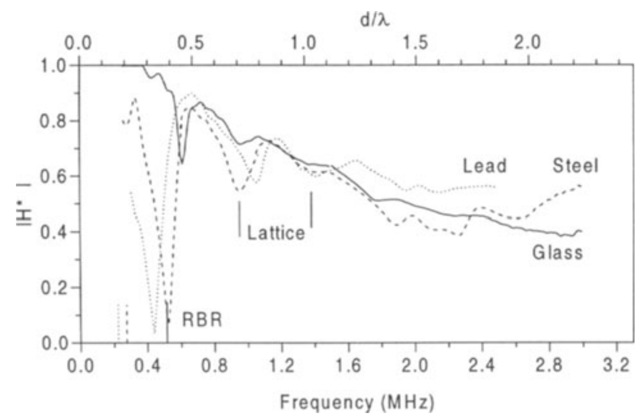


Figure 5. Magnitude of through-transmission transfer function for rectangular periodic arrangement with 1.86 mm spacing (b).

body resonance at 0.2 MHz and 0.3 MHz respectively. Because of low ultrasonic velocity in lead, the sample with a layer of lead particles shows a large number of resonances within 3 MHz frequency range. Several random distributions with the same area fraction were tested yielding nearly identical magnitude and phase results.

Periodic Distribution of Inclusions

The magnitude of the transfer function through a periodic distribution of inclusions is presented in Figures 5, 6, and 7, for the particles arrangement shown in Fig. 1 b, d and e respectively. The minima in the magnitude of the transfer function for the periodic layer occur approximately at the same frequencies for all three types of particles with the exception of strong minimum at low frequency and minimum at 2.3 MHz for samples with steel particles which corresponds to the shape resonance of a steel ball (see Fig. 3).

We conjecture that these minima are due to the periodicity of the inclusions. Minima of a transfer function in Fig. 5 at 0.95 MHz, in Fig. 6 at 1.1 MHz, and 2.2 MHz, in Fig. 7 at 0.95 MHz and 1.9 MHz correspond to an integral number of wavelengths between two neighboring particles. The minimum at 1.5 MHz in Fig. 6 and the corresponding minimum

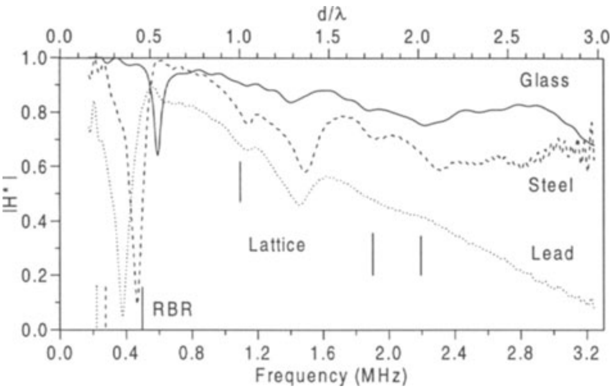


Figure 6. Magnitude of through-transmission transfer function for hexagonal periodic arrangement with 2.28 mm spacing(d).

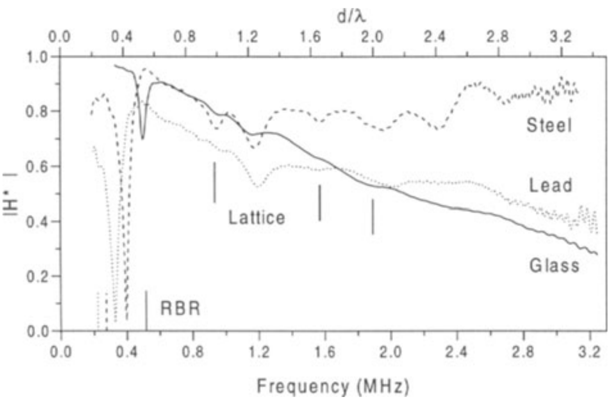


Figure 7. Magnitude of through-transmission transfer function for hexagonal periodic arrangement with 2.63 mm spacing(e).

at 1.2 MHz in Fig. 7 are probably due to an interference of the incident wave and the wave that is speculary reflected by one particle in the direction of the neighboring particle and then back in the direction of the receiving transducer. Minimum at 0.95 MHz and 1.9 MHz in Fig. 5 for square particle arrangement correspond to an integral number of wavelengths fitted along the diagonal of the square cell. In the case of the steel particles, an additional minimum is observed at 2.3 MHz, which corresponds to the excitation of the particle resonance.

Position of the low frequency resonance minimum depend on both inter particle spacing and particle properties. Particle density strongly affects both resonance depth and frequency. The frequency of this minimum is approximately inversely proportional to the particle spacing and corresponds to the integer number of shear wavelength between particles. We speculate that rigid body resonance [15] is involved in this phenomenon. If the frequency of the rigid body resonance is happens to be close to the lattice resonance frequency, the through-transmission transfer function exhibits a deep (up to 40 dB) resonant minimum. Analogous behavior of the regular array of spherical bubbles in liquid where amplification of giant bubble resonance occurs is well known [16].

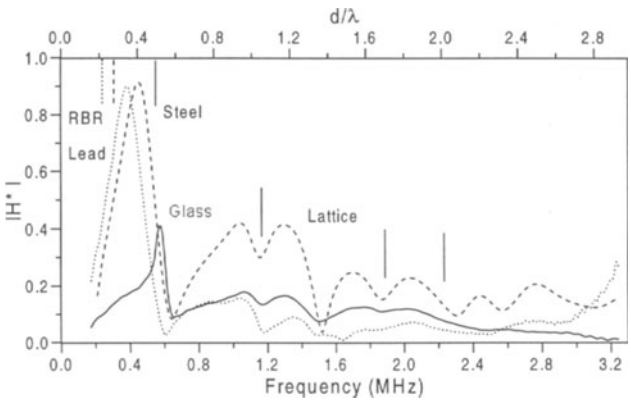


Figure 8. Magnitude of reflection transfer function for periodic arrangement (d).

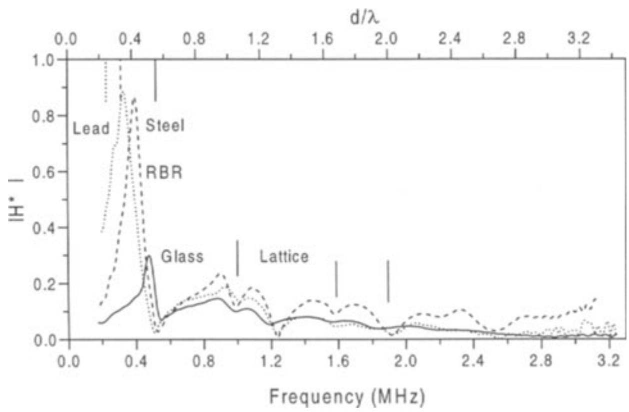


Figure 9. Magnitude of reflection transfer function for periodic arrangement (e).

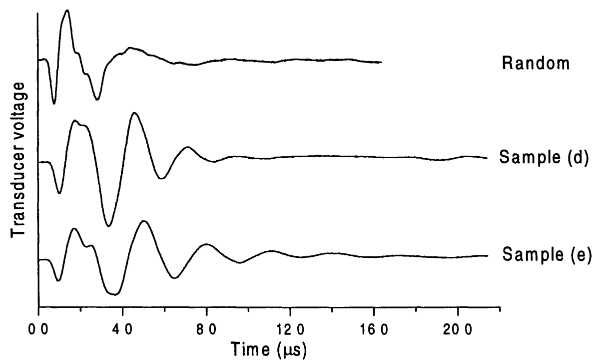


Figure 10. Time domain reflected signal from periodic single layer of lead particles.

Reflection transfer function for 2.28 and 2.63 mm hexagonal particle arrangement is presented in Fig. 8 and Fig. 9. Again, the minima in the magnitude of the transfer function occur approximately at the same frequencies for all three types of particles with the exception of a strong maximum at low frequency where magnitude of a transfer function nearly reach unity.

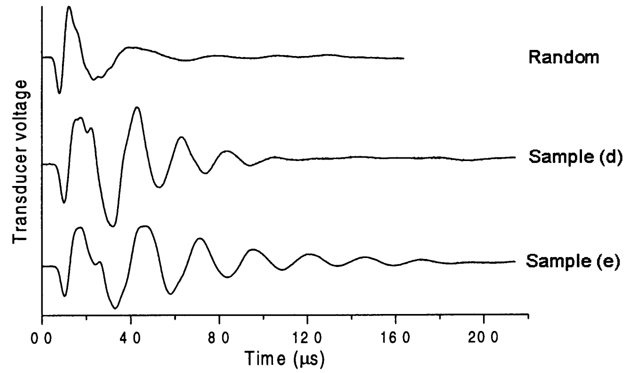


Figure 11. Time domain reflected signal from periodic single layer of steel particles.

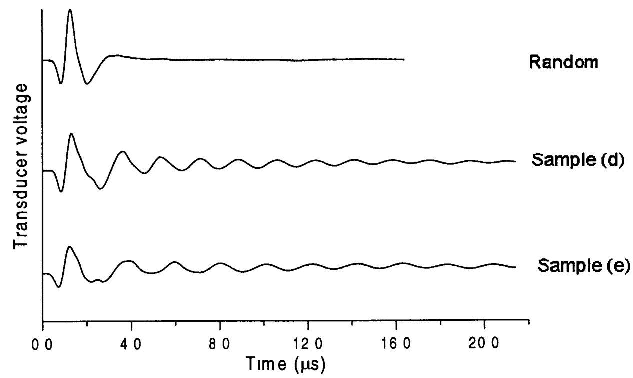


Figure 12. Time domain reflected signal from periodic single layer of glass particles.

A reflected ultrasonic impulses from periodic layer of lead, steel and glass particles with 2.28 mm and 2.63 mm spacing and random layer with 0.24 area fraction are presented in time domain in Fig. 10 through Fig. 12 respectively. The reflected signal for the layer with random distribution of particles looks similar to that transmitted through the neat specimen with some small but visible distortion in case of lead particles. In contrast, signal reflected from periodic layer indicate monochromatic oscillations with gradual, approximately exponential amplitude decay. The frequency of these oscillations corresponds to the low frequency maximum of the reflection transfer function on Figures 8 and 9. Similar effect has been found earlier [6] in through transmission for three dimensional periodic particular composites.

The peak amplitude of the reflected signal is smaller for the light glass particles than for the heavier lead and steel particles. The oscillation damping factor decrease with decreasing of particle density from lead to glass and with increasing of particle spacing from 2.28 mm to 2.63 mm.

## CONCLUSION

For a single layer of inclusions, we have shown that the arrangement of the inclusions has a significant effect on a wave propagating normal to the layer. The influence of particle material properties and spacing on wave propagation was studied in great detail.

Layers with both a random and a periodic distribution of particles have a transfer function with several minima which can be attributed to the resonances of the individual particles. A transfer function for a layer with a periodic particle distribution has additional sharp minima; the frequency of some of these additional minima is inversely proportional to the particle spacing and does not depend on particle acoustical properties. If this frequency happens to be close to the rigid body resonance of a particle, the through-transmission transfer function exhibits a strong (up to 40 dB) resonance minimum with the depth and position depending on both the inclusion density and the particle spacing. The acoustic response of a particle layer on an impulse excitation consists of steady oscillation with weak exponential decay, indicating the presence of a mechanism of storing vibration energy within the layer.

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